DEVELOPMENTS IN THE MODELLING OF FUSE BREAKING TESTS

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Summary

The paper describes a number of improvements to the modelling of short-circuit tests on current-limiting fuses with multiple parallel notched elements in a granular filling medium. Some of these improvements are concerned with the physical models while others are concerned with the numerical methods required to obtain fast and accurate solutions. Some typical results are given, and the need for improvements to the test-plant model is discussed.

1. Introduction

In 1980 Gnanalingam and Wilkins described a method for computing the complete current and voltage transients for a short-circuit test on a current-limiting fuse [1]. The method used a finite-difference method to calculate the transient temperature distribution during the prearcing period, and the arcing period was modelled using a quasi-static model of the arcs in the notch zones, which expanded radially due to fusion of the filler, and axially due to burnback. The method was based upon experimental work on silver fuse elements with uniformly spaced rectangular notches, in a particular type of quartz sand [2].

The arcing models required a number of non-linear ordinary differential equations need to be solved. During each integration time step the arc length increases due to burnback, and this was allowed for by creating a new "lumen segment" to represent the additional part of the arc. The cross-sectional area of the new lumen segment was then added to the list of variables to be integrated, and so the order of the model increased after each time step.

Subsequent work by Daalder and Schreurs [3], Leistad *et al* [4], and Eger and Rother [5], has confirmed that this type of fuse model gives a realistic representation of the breaking process.

An interactive program based upon this model was described at Trondheim in 1984 [6].

In this paper a number of improvements to the basic model of 1980 are described. These improvements can be grouped into three categories as follows :

(a) Improvements to the physical models. These are almost all connected with the arcing phase, as the physical basis of the prearcing phase is well understood and documented.

(b) Improvements to the computational algorithms. These are needed to improve the flexibility, speed and accuracy of the computations to enable the user to model a much wider range of fuse designs.

(c) Extension of the range of materials which can be specified for the element and filler. Whilst the standard thermophysical properties of most materials are known, some of the properties used in the arcing models need to be estimated from controlled experiments.

Some typical results obtained with the improved models are given, and the accuracy, usage, and possible future developments are discussed.

2. Improvements to prearcing computations

It is desirable for the program user to be able to select the fuse element design very easily, using a menu to choose the shapes of the reduced sections and their axial location along the fuse element. When running the program the required finite-difference meshes need to be generated automatically, with the mesh sizes reduced in regions of high current density, to give higher accuracy.

2.1 Solution method

The program described in 1980 used the Crank-Nicholson formulation of the finite-difference methods, with the new element temperature distributions being computed by iteration after each time step. Experience has shown that the fully implicit formulation is preferable for general-purpose use, with direct rather than iterative solution. When sudden changes occur, for example in the solution time-step, stable oscillations are generated if the Crank-Nicholson method is used. These oscillations are well-known [7], and can be sometimes be troublesome. No such oscillations occur with the fully implicit method, which has been found, to be more robust for general-purpose use. (Similar considerations apply if finite-element methods are used for the discretisation of the space variables [8]. Again a fully implicit formulation for the time-derivative is preferable).

A linear set of equations for the temperatures at the nodal points on the fuse elements has to be solved at each time step [6], for each notch design, and for the plain element sections between the notch zones. These equations are most efficiently solved using sparse-matrix methods, with triangular factorisation and back-substitution [9]. The mesh-generation algorithms need to generate a node numbering scheme which is near-optimal, which gives a dramatic reduction in solution time. The equations only need to be retriangularised when a change in time step occurs.

2.2 Time step control

A general purpose program must incorporate schemes to automatically set the initial solution time step Δt, and to subsequently alter Δt as necessary. If the accuracy of the solution is too low the time step must be reduced, while if the accuracy is unnecessarily high, it must must be increased, to give acceptably fast solutions. Experience has shown that this can be achieved, as far as the element temperatures are concerned, by reducing the time step if the maximum increment in the temperature of any node is about to exceed $\Delta T_m/50$ and increasing it if the maximum increment falls below $\Delta T_m/80$, where ΔT_m is the temperature rise required to reach the fusion point from the initial ambient value.

However with notches of different styles on the same element the situation occurs that arcing is in progress at some notch zones while others have not yet melted. So the above scheme for time step control needs to be combined with one suitable for the arcing variables. This combined scheme will be discussed later.

3. Improvements to arcing models

Although the basic concept of the arcing process is unchanged, some substantial improvements to the detailed physical models have been made.

3.1 Arc quasi-static characteristic

Gnanalingam's work [2] was principally concerned with the high-current end of the arc characteristic, and in [1] a very rough correction was made to allow for the fact that the slope of the characteristic becomes negative as the current falls. More recent work [10] has extended the theoretical model of the quasi-static characteristic into the low-current zone, which gives improved modelling of the behaviour during the later stages of arcing.

3.2 Characterisation of different fillers

During arcing radial expansion of the arc channel is determined by the input power from the arc and the enthalpy required to heat the filler to the effective fusion temperature [1],[3]. However, this model alone is not sufficient to explain the differences between fillers in their ability to control the arc. For example, the radial expansion of an arc in bonded sand is very much lower than in loose sand, despite the fact that the required enthalpy differs very little for the two materials. This must be attributed to differences in the viscous flow properties of the liquid filler [3]. It appears that the flow into bonded sand is much lower, and this effect can be taken into account by multiplying the lumen expansion rate by a flow coefficient, determined experimentally for each type of sand. Loose sands have flow coefficients around 1.0, while with bonded sand, values as low as 0.2 are found.

3.3 Burnback rate

The rate of burnback used in [1] was obtained from an empirical power-law, derived from experiments on the burnback of silver strips in quartz sand [2]. Subsequently Daalder developed a model for the burnback process which included the effect of the temperature of the strip ahead of the arc root [3]. Although the difference in results is small, Daalder's model has a more sound physical basis and should be used in preference to the power-law formula. It has however been found desirable to make an adjustment for the effect of the filler material. During burnback, element metal is removed principally in liquid form which is forced away from the arcing zone through the gaps between the sand grains. Thus the porosity of the sand to this flow has an influence on the burnback rate.

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3.4 Effect of sand volume

It is well known that the arcing I^2t for a given element design falls significantly if the inner diameter of the fuse tube is reduced [11]. During arcing the sand is compressed due to the pressure exerted by the arc column, resulting in an additional increase in the lumen cross-section. Pastors [12] has demonstrated that the arcing I^2t can be reduced by application of external pressure from a piston on the sand grains (although the intergranular gas pressure remains atmospheric).

If the arc column pressure is p, the increase in lumen cross-section will be given by pA_s/K, where A_s is the total sand cross-section per unit length, and K is the compression modulus of the sand. Thus if the sand volume A_s is reduced, the increase in section will fall, resulting in a higher arc voltage gradient and a lower arcing I^2t.

This gives a very simple model for correcting for the effect of the inner diameter of the fuse tube, by adding a small time-dependent area to the lumen cross-section during solution.

3.5 Initial arc voltage

When the centre of a notch zone reaches its melting point a short arc is formed, and for rectangular notch the initial arc length can be taken as equal to the notch length [1]. However, with other other notch profiles, for example semicircular, there is some doubt as to the the value to be assumed for the initial arc length. One possibility is to assume a length of zero and then allow the arc to extend along the notch zone using the usual burnback formula. However the notch disintegration is explosive and an initial arc length greater than zero is more realistic. A good compromise is to set the length to the value at which the element cross-sectional area is 20% larger than the minimum at the centre of the notch, although this value is not critical.

A more important effect with semicircular or tapered notches during the early stages of arcing appears to be a more rapid rate of lumen expansion due to the very low quantity of entrapped liquid metal from the element in the adjacent fulgurite wall. To quote from Daalder " ... It seems that metal droplets increase the cohesion between the sand grains and improve, by a kind of glueing effect, the solidity of the wall around the arc." [3]. This does indeed appear to be the case. Using this concept a correction to the lumen expansion rate can be made to allow for the low density of entrapped metal in the notch zones, and this has been found to give better results for semicircular and similar notch designs.

4. Automatic control of solution time step

The arcing models give rise to a non-linear set of ordinary differential equations, in which the principal state variables are the circuit current, the lengths of the arcs in the notch zones, and the cross-sectional areas of the arc lumen segments. There are also auxiliary variables such as arc energy and I^2t which can be conveniently added to the set [1]. During some parts of the arcing period these variables change rapidly, and a small time step is needed to follow them, while at other times, usually during the later stages of arcing, a much larger time step is possible. Runge-Kutta integration with automatic adjustment of the time step to achieve a preset accuracy [13] has been found to be most convenient. In this method the equations are first solved using 3rd-order R-K integration, and then the solution is repeated using a more accurate 4th-order method. The difference between the two results is used to estimate the maximum truncation error and if this is greater than a preset tolerance, the time step is halved and the calculations are redone. If during computation the truncation error falls below one-tenth of the tolerance, the time step is increased by 50% at the succeeding integration step.

Although this scheme works well for controlling the solution for the arcing variables, it is insufficient for checking on the accuracy of the transient heating calculations as described in section 2.2. One possible way to do this is to reformulate the finite-difference equations as a set of linear o.d.e.'s [7], add these to the list of state variables, and to use R-K integration to solve for all the variables, including temperature rises, simultaneously. This method has been tried but it is not efficient. Open-type integration schemes such as R-K require the time step to be much smaller than the smallest time constant in the set of equations [13]. If the heating calculations are included, the solution time step becomes (unnecessarily) dictated by the thermal time constants of the notch zones, yielding a very small time step and unacceptably long solution times.

It is much better to use the direct implicit method described in section 2 to calculate the element temperature distribution and to use a combined scheme for controlling the time step. A typical method is given below. (Omitting model switching and associated logic).

(i) Integrate the non-linear set for the new state variables, reducing the time step if necessary to ensure accuracy. This will yield, among other things, the new circuit current.

(ii) Calculate the new temperature distributions for the notch zones not yet arcing. If the maximum temperature increment exceeds $\Delta Tm/50$, reset all state variables and temperatures to "old" values and return to step (i).

(iii) Increment the time variable.

(iv) If the truncation error from the R-K exceeds one-tenth of the required tolerance, leave Δt unchanged. Otherwise increase the time step as follows :

  (a) If all notch zones are arcing increase Δt by 50%

  (b) Otherwise increase Δt by an amount which by extrapolation is expected to give a maximum temperature increment of $\Delta Tm/80$ on the next step

(v) Go to step (i).

This scheme gives fast efficient solutions for all fuse designs encountered to date.

5. Other computational techniques

5.1 Switching of models and logical control

During the numerical solutions described above to obtain the transient temperatures and arcing variables, there are many stages at which the models need to be changed, enabled or disabled, consequent upon the occurrence of some "event". Typical events include fusion of the notch zones, merging of arcs between adjacent notch zones, arcs reaching the end-caps, commutation of arc current between parallel elements, arc extinction, and so on. In a fuse with several different notch designs, some notch zones may be arcing while some are not, and the merging of adjacent arcs may take place at different points on the elements at different times. It is a very complex job to keep track of the state of all parts of the total system model. It is necessary to have a formally-structured procedure for doing this, firstly to ensure reliability, and secondly to enable upgrades to be added to the models with the minimum of difficulty.

The basic procedure is to have a set of logical variables describing the states and sub-states of the model, and at the end of each integration time step, logical tests are made to see whether a change of state has occurred during the step. If so, the time at which the change occurred (t') is calculated by linear interpolation, the state(s) are switched, and the values of all numeric solution quantities are reset, by interpolation, to their values at time t' before resuming solution.

This basic procedure is not enough, however, as sometimes more than one state change can occur during a time step. Thus it is necessary first to record all state changes which occur and then to determine which of these changes occurred earliest, i.e. with the lowest value of t'. The model is then switched to this point before resuming, which ensures that the changes of state occur in the right order. In some cases this can be an important consideration [10].

Within this scheme, however, special consideration needs to be given to the case of merging of arcs between adjacent notch zones. Whilst in general each notch zone needs to be treated individually, it is very common to have a uniform spacing of notches, in which case merging takes place almost simultaneously along the length of the element. For a high-voltage fuse there may be a hundred or so series notches, and if they are uniformly spaced, the above solution procedure would reset the models a hundred or so times during the time step when merging occurred, in an order which depended upon the rounding error in the solution for the arc lengths. To give a more efficient procedure, the states should be reset after logical testing for all arcs about to merge whose lengths do not differ by more than 1%. This gives a much more rapid progress through the transition with negligible change in accuracy.

5.2 Addition of new lumen segments

In the original scheme [1], a new axial lumen segment was added to each arc at every time step. During the later stages of arcing, when the arc current becomes low, these new segments are very short, because the burnback velocity is very low. In these cases a large reduction in computing time can be achieved if, instead of creating a new lumen segment, the extra arc length is simply added to the previous segment. This drastically reduces the number of state variables generated during solution. Experience has shown that if this procedure is adopted when the extra length is less than about 1% of the total arc length, there is negligible difference in the results obtained.

6. Software design

6.1 User interface

It should be emphasised that the models and computational algorithms described in this paper must be transparent to the software user, who is interested in the final results and not usually in the means used to achieve them. Developing a simple system for the input of fuse designs and the output of results requires considerable effort but the ever-increasing availability of cheap computer power makes large improvements possible in this area.

6.2 Structured design

With increased complexity of the models and algorithms, and with a modern user interface, structured design of software is essential. All significant software functions should be contained within modules which are planned to combine together in a way that allows for future alterations and additions. Thus if it is decided that an improved burnback model is to be adopted, it must be possible to do this very simply, by changing one program module, with little or no alteration to the remaining modules.

7. Typical results

Fig.1 shows a comparison between the measured breaking transients and the values computed using the improved models and algorithms described above for a 600A fuse tested nominally at 600V 100kA. The curves show a prearcing period followed by a small voltage increase when one of the notches begins arcing. Somewhat later the remaining notches, which have larger widths, melt and the fuse voltage increases further, followed by a rapid rise in voltage as all arcs burn back. Later, as the current falls and the arcs merge into one (in stages) the fuse voltage falls along with the current until arc extinction occurs.

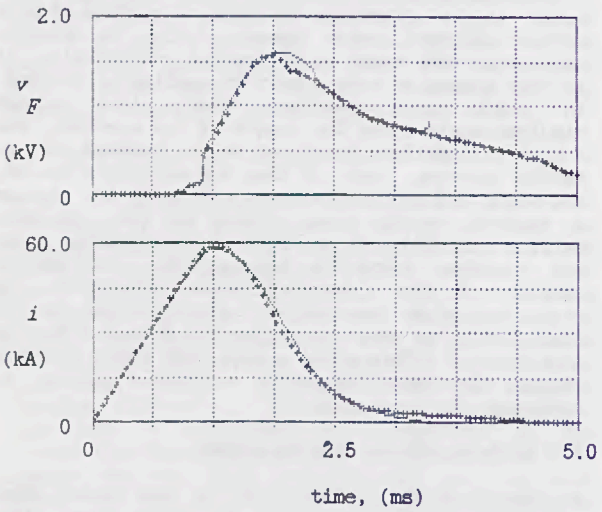

Fig. 1 Short-circuit transients for 600A fuse at 600V, 100kA nominal

———————— *computed*

+ + + + + + + + + + *test results*

Using the improved models and algorithms, with a variety of fuse designs and filling media, peak let-through currents can usually be computed within 5%, while peak arc voltage, arc energy, and I^2t are usually within 15%.

The results for unbonded sand are better than for bonded sand, and so there is scope for improving the models of the arcing process within bonded sand.

8. Test plant model

Apart from the modelling of arcing in bonded sand, there is little scope for improvements to the fuse models described above. If further improvement in accuracy is required, a better model of the test circuit is needed. The results shown in Fig.1 were obtained by representing the source circuit in the usual way, by a fixed R-L series circuit, determined from the nominal test conditions, i.e. voltage, prospective current, and power factor. However recent experience has shown that this model is inadequate if high accuracy is required.

In most laboratories the short-circuit current is obtained from a synchronous generator, the inductance of which is time-varying. The total circuit inductance consists of the generator inductance plus the inductance external to the generator and is therefore not constant. Studies have shown that this can typically cause an error of 3-4% in the computed value of I^2t.

However a much more important consideration is that the test circuit resistance is not constant. The estimate of R usually used is based upon a power-factor value derived from a calibration shot. It is obtained from the d.c. decrement one half-cycle after switch-on, i.e. after 10ms for 50 Hz, and this gives a measure of the d.c. resistance of the test circuit. However most fuse breaking tests are over within a few milliseconds, and during this period the transient resistance of the circuit is much higher than the d.c. value, because of the transient skin and proximity effects and transients induced in other conductive parts external to the test circuit. The power for these losses must be met from the test circuit and they are made manifest by an transient increase in the test circuit resistance.

The above errors in the test-circuit model give rise to errors of the same order as those due to the fuse model. Significant further improvement is only possible if progress in the modelling of the test plant, as well as the fuse, can be made.

9. Conclusions

The paper has described a number of extensions and improvements to the modelling of fuse breaking tests. For computing transient element temperatures a fully implicit formulation is recommended, with direct solution using sparse-matrix methods. Improvements to the arcing models include extension to the quasi-static arc characteristic, representation of the flow properties of different fillers under arcing, improved burnback models, and a method for representing the effect of the inner diameter of the fuse tube.

Automatic control of the solution time step is needed to obtain fast and accurate results, and a suitable method has been described, which combines the differing requirements of the prearcing and arcing models. A structured scheme for model switching and logical control is needed for reliability and extensibility. The improved models have been tested with a variety of different fuse designs and give results which are close to those obtained by short-circuit tests, but further improvement requires a simultaneous improvement in the modelling of the short-circuit test plant.

10. References

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